



**Plainfield Public Schools
Mathematics
Unit Planning Organizer**

Grade/Course	Algebra 2
Unit of Study	Modeling with Rational Exponents
Pacing	9 weeks

<u>Standard for Mathematical Practices</u>	
MP1.	Make sense of problems and persevere in solving them.
MP2.	Reason abstractly and quantitatively.
MP3.	Construct viable arguments and critique the reasoning of others.
MP4.	Model with mathematics.
MP5.	Use appropriate tools strategically.
MP6.	Attend to precision.
MP7.	Look for and make use of structure.
MP8.	Look for and express regularity in repeated reasoning.

Hyperlinks are noted underlined in italics

UNIT STANDARDS

N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.RN.1 Explain how the definition of the meaning of rational exponents those values, allowing for a notation for radicals in terms of rational follows from extending the properties of integer exponents to exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(1/3)^3$ to hold, so $(5^{1/3})^3$ must equal 5*

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B.4.B Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★*

Hyperlinks are noted underlined in italics

A.SSE.B.3.C Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A.SSE. B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *

F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.4 Understand the inverse relationship between exponents and logarithms. *For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.*

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

F.IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

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“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD:		
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(1/3)^3$ to hold, so $(5^{1/3})^3$ must equal 5.</i>		
Explain	rational exponents	1

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD:		
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.		
Rewrite	radicals and rational exponents	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
SUPPORTING STANDARD:		
A.REI.B.4.B Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.		
Solve	Quadratic equations	2

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PPS Secondary Math Curriculum Algebra 2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: A.SSE.3 Write equivalent expressions for exponential functions using the properties of exponents.		
Write	exponential functions	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: A.SSE.B.3.C Use the properties of exponents to transform expressions for exponential functions. <i>For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>		
Use	Exponential function	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity</i>		
Interpret sketch	Function graphs	3 2

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PPS Secondary Math Curriculum Algebra 2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
SUPPORTING STANDARD: F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.		
Write	Function	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
SUPPORTING STANDARD F.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.		
Use	exponents	2

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PPS Secondary Math Curriculum Algebra 2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: A.SSE. B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*		
Derive	Geometric Series	4

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *		
Write	Arithmetic sequences Geometric series	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
FOCUS STANDARD: F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).		
Construct	Linear functions Exponentials functions	4

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PPS Secondary Math Curriculum Algebra 2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
<p align="center">FOCUS STANDARD:</p> <p>F.LE.A.4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.</p>		
Understand	Exponents Logarithms	1

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
<p align="center">ADDITIONAL STANDARD:</p> <p>F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.</p>		
Interpret	Linear Functions Exponential Functions	4

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“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
ADDITIONAL STANDARD: A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.		
Solve	Systems of linear equations	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
ADDITIONAL STANDARD: A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.		
Solve	Linear equation	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
ADDITIONAL STANDARD: N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.		
Know	Complex number	1

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PPS Secondary Math Curriculum Algebra 2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
ADDITIONAL STANDARD:		
N.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.		
Use	Complex number	2

“Unwrapped” Skills (students need to be able to do)	“Unwrapped” Concepts (students need to know)	DOK Levels
ADDITIONAL STANDARD:		
N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.		
Solve	Quadratic equations	2

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New Jersey Student Learning Standards and Mathematical Practices Examples and Explanations

Number and Quantity: The Complex Number System (N-CN)		
Perform arithmetic operations with complex numbers.		
<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
HS.N-CN.A.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	<i>HS.MP.2.</i> Reason abstractly and quantitatively. <i>HS.MP.6.</i> Attend to precision.	

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PPS Secondary Math Curriculum Algebra 2

<p>HS.N-CN.A.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p>	<p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.7.</i> Look for and make use of structure.</p>	<p>Example:</p> <ul style="list-style-type: none"> Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3 - 2i)(-7 + 4i)$ <p>Solutions may vary; one solution follows:</p> $(3 - 2i)(-7 + 4i)$ $3(-7 + 4i) - 2i(-7 + 4i) \text{ Distributive Property}$ $-21 + 12i + 14i - 8i^2 \text{ Distributive Property}$ $-21 + (12i + 14i) - 8i^2 \text{ Associative Property}$ $-21 + i(12 + 14) - 8i^2 \text{ Distributive Property}$ $-21 + 26i - 8i^2 \text{ Computation}$ $-21 + 26i - 8(-1) \quad i^2 = -1$ $-21 + 26i + 8 \text{ Computation}$ $-21 + 8 + 26i \text{ Commutative Property}$ $-13 + 26i \text{ Computation}$
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Number and Quantity: The Complex Number System (N-CN)

Use complex numbers in polynomial identities and equations.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
HS.N-CN.C.7. Solve quadratic equations with real coefficients that have complex solutions.		Examples: <ul style="list-style-type: none"> • Within which number system can $x^2 = -2$ be solved? Explain how you know. • Solve $x^2 + 2x + 2 = 0$ over the complex numbers. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$.

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve equations and inequalities in one variable.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>									
HS.A-REI.B.4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	<i>HS.MP.2.</i> Reason abstractly and quantitatively. <i>HS.MP.7.</i> Look for and make use of structure. <i>HS.MP.8.</i> Look for and express regularity in repeated reasoning.	Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$. <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Value of Discriminant</th> <th>Nature of Roots</th> <th>Nature of Graph</th> </tr> </thead> <tbody> <tr> <td>$b^2 - 4ac = 0$</td> <td>1 real roots</td> <td>intersects x-axis once</td> </tr> <tr> <td>$b^2 - 4ac > 0$</td> <td>2 real roots</td> <td>intersects x-axis twice</td> </tr> </tbody> </table>	Value of Discriminant	Nature of Roots	Nature of Graph	$b^2 - 4ac = 0$	1 real roots	intersects x-axis once	$b^2 - 4ac > 0$	2 real roots	intersects x-axis twice
Value of Discriminant	Nature of Roots	Nature of Graph									
$b^2 - 4ac = 0$	1 real roots	intersects x-axis once									
$b^2 - 4ac > 0$	2 real roots	intersects x-axis twice									

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PPS Secondary Math Curriculum Algebra 2

<p>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p>		$b^2 - 4ac < 0$	2 complex roots	does not intersect x-axis
<p>Examples:</p> <ul style="list-style-type: none"> • Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation. • What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related? 				

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Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve systems of equations.

<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p>HS.A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>	<p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> <p><i>HS.MP.6.</i> Attend to precision.</p> <p><i>HS.MP.7.</i> Look for and make use of structure.</p> <p><i>HS.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.</p> <p>Examples:</p> <ul style="list-style-type: none"> José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system. <div style="text-align: center;"> </div> <ul style="list-style-type: none"> Solve the system of equations: $x + y = 11$ and $3x - y = 5$. Use a second method to check your answer. Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$. The opera theater contains 1,200 seats, with three different prices. The seats cost \$45 per seat, \$50 per seat, and \$60 per seat. The opera needs to gross \$63,750 on seat sales. There are twice as many \$60 seats as \$45 seats. How many seats in each level need to be sold?

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PPS Secondary Math Curriculum Algebra 2

<p>HS.A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i></p>	<p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> <p><i>HS.MP.6.</i> Attend to precision.</p> <p><i>HS.MP.7.</i> Look for and make use of structure.</p> <p><i>HS.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>Example:</p> <ul style="list-style-type: none">Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (d) that her car travels as a function of time in hours (t) since Suzette's car passed is given by $d = 3500t^2$. <p>Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette.</p>
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PPS Secondary Math Curriculum Algebra 2

Standards	Mathematical Practices	Explanations and Examples
<p><i>Students are expected to:</i></p> <p>HS.N-RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</i></p>	<p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p>	<p>Students may explain orally or in written format.</p>
<p>HS. N-RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p>	<p><i>HS.MP.7.</i> Look for and make use of structure.</p>	<p>Examples:</p> <ul style="list-style-type: none"> • $\sqrt[3]{5^2} = 5^{\frac{2}{3}} ; 5^{\frac{2}{3}} = \sqrt[3]{5^2}$ • Rewrite using fractional exponents: $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ • Rewrite $\frac{\sqrt{x}}{x^2}$ in at least three alternate forms. $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}} = \frac{1}{x\sqrt{x}}$ <p>Solution:</p> <ul style="list-style-type: none"> • Rewrite $\sqrt[4]{2^{-4}}$ using only rational exponents.

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		<ul style="list-style-type: none"> Rewrite $\sqrt[3]{x^3 + 3x^2 + 3x + 1}$ in simplest form.
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Functions: Building Functions (F-BF)

Build a function that models a relationship between two quantities.

<p>HS.F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p>	<p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> <p><i>HS.MP.8.</i> Look for and express regularity in repeated reasoning.</p>	<p>An explicit rule for the nth term of a sequence gives a_n as an expression in the term's position n; a recursive rule gives the first term of a sequence, and a recursive equation relates a_n to the preceding term(s). Both methods of presenting a sequence describe a_n as a function of n.</p> <p>Examples:</p> <ul style="list-style-type: none"> Generate the 5th-11th terms of a sequence if $A_1 = 2$ and $A_{(n+1)} = (A_n)^2 - 1$ Use the formula: $A_n = A_1 + d(n - 1)$ where d is the common difference to generate a sequence whose first three terms are: -7, -4, and -1. There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population. Given the formula $A_n = 2n - 1$, find the 17th term of the sequence. What is the 9th term in the sequence 3, 5, 7, 9, ...? Given $a_1 = 4$ and $a_n = a_{n-1} + 3$, write the explicit formula.
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PPS Secondary Math Curriculum Algebra 2

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

HS.F-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

HS.MP.4. Model with mathematics.

HS.MP.8. Look for and express regularity in repeated reasoning.

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples:

- Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.

x	$f(x)$
0	1
1	3
3	27

- Sara's starting salary is \$32,500. Each year she receives a \$700 raise. Write a sequence in explicit form to describe the situation.

Hyperlinks are noted underlined in italics

PPS Secondary Math Curriculum Algebra 2

<p>HS.F.LE.A.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.</p>	<p><i>HS.MP.7.</i> Look for and make use of structure.</p>	<p>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms.</p> <p>Example:</p> <ul style="list-style-type: none">• Solve $200 e^{0.04t} = 450$ for t. <p>Solution:</p> <p>We first isolate the exponential part by dividing both sides of the equation by 200.</p> $e^{0.04t} = 2.25$ <p>Now we take the natural logarithm of both sides.</p> $\ln e^{0.04t} = \ln 2.25$ <p>The left hand side simplifies to $0.04t$, by logarithmic identity 1.</p> $0.04t = \ln 2.25$ <p>Lastly, divide both sides by 0.04.</p> $t = \ln (2.25) / 0.04$ $t \approx 20.3$
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<u>Standards</u> <i>Students are expected to:</i>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
HS.F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.	<i>HS.MP.2.</i> Reason abstractly and quantitatively. <i>HS.MP.4.</i> Model with mathematics.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. Example: <ul style="list-style-type: none">• A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.

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PPS Secondary Math Curriculum Algebra 2

Algebra: Seeing Structure in Expressions (A-SSE)

Write expressions in equivalent forms to solve problems.

<u>Standards</u>	<u>Mathematical Practices</u>	<u>Explanations and Examples</u>
<p><i>Students are expected to:</i></p> <p>HS.A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p>	<p><i>HS.MP.1.</i> Make sense of problems and persevere in solving them.</p> <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p>	<p>Students will use the properties of operations to create equivalent expressions.</p> <p>Examples:</p> <ul style="list-style-type: none"> Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. Write the expression below as constant times a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger.
<p>c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p>		$o \quad \frac{(2x^3)^2(3x^4)}{(x^2)^3}$

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Essential QuestionsCorresponding Big Ideas

Essential Questions	Corresponding Big Ideas
<p>How can the properties of the real number system be useful when working with polynomials and rational expressions?</p> <p>What components are needed to graph a rational function and how are they found?</p> <p>How can we expression a radical expression?</p> <p>How do you model a quantity that changes regularly over time by the same percentage?</p>	<p>Functions apply to a wide range of situations. They do not have to be described in any specific expression or follow a regular pattern. They apply to cases other than those of “continuous variation”. For example, sequences are functions.</p> <p>For functions that map real numbers to real numbers, certain patterns of covariation, or patterns in how two variables changes together, indicate membership in a particular family of functions and determine the type of formula that has the function has.</p> <p>A rate of change describes how one variables quantity changes with respect to another -in other words, a rat of change describes the covariation between two variables.</p> <p>A function’s rate change is one of the main characteristic that determine what kinds of real whole phenomena the function can model.</p> <p>Quadratic functions are characterized by a linear rate of change of the rate change (the second derivative) of a quadratic function is constant. Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic are real, they are symmetric about the x - coordinate of the maximum or minimum point.</p>

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	<p>Exponential Function are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increases by 1 unit, the output is multiplied by constant factor. Exponential functions connects multiplication to addition through the equation $a^{b+c}=(a^b)(a^c)$</p> <p>For functions that map the real numbers to the real numbers, composing a functions with “shifting” or scaling” functions changes the formula and graph of the functions in readily predictable ways.</p> <p>Under appropriate conditions, functions have inverses</p> <p>Functions can be represented in various ways, including through algebraic means (e.g., equation), graphs, word descriptions, and tables.</p> <p>Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristic, and some may show only part of the function.</p> <p>Some representations of a function may be more useful than others, depending on the context.</p> <p>Links between algebraic and graphical representations of a functions are especially important in the studying relationship and change.</p> <p><i>Sources:</i> <i>Cooney, T & Beckman, Sybilla. (2010). Developing essential understanding of Functions 9-12. Reston, VA: The National Council of Teachers of Mathematics, Inc</i></p>
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Student Learning Objectives

Student Learning Objectives	Concepts/Skills		Mathematical Practices
Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties N.CN.A.1, N.CN.A.2	<p>Concepts:</p> <ul style="list-style-type: none"> Complex number i is defined such that $i^2 = -1$. Every complex number has the form $a + bi$ with a and b real. <p>Students are able to:</p> <ul style="list-style-type: none"> $i^2 = -1$ and the commutative, associative properties to add and subtract complex numbers are to be used. Determine that $i^2 = -1$ and the commutative, associative, and distributive properties to multiply complex numbers. 		MP.6 MP.7
Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring. N.CN.C.7, A.REI.B4	<p>Concepts:</p> <ul style="list-style-type: none"> As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square. <p>Students are able to:</p>	Tasks are limited to equations with non-real solutions.	MP.5

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	<ul style="list-style-type: none"> • Solve quadratic equations in one variable that have complex solutions by taking square roots. • Solve a quadratic equation in one variable that have complex solutions by completing the square. • Solve a quadratic equation in one variable that have complex solutions by factoring. • write complex solutions in $a \pm bi$ form. 		
<p>Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.A.REI.C.7</p>	<p>Concepts:</p> <ul style="list-style-type: none"> • Solutions of linear systems contain different function types. <p>Students are able to:</p> <ul style="list-style-type: none"> • Solve a system containing one linear equation and one quadratic equation algebraically. • Graph a system containing one linear equation and one quadratic equation to determine a solution. 	<ul style="list-style-type: none"> • Tasks have thin context or no context. 	<p>MP1</p>

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<p>Solve algebraically a system of three linear equations. A.REI.C.6</p>	<p>Concepts:</p> <ul style="list-style-type: none"> Solving a system of linear equations containing n variables requires n equations. <p>Students are able to:</p> <ul style="list-style-type: none"> Use the substitution method and/or elimination method to find the solution of a system containing three linear equations. 	<ul style="list-style-type: none"> Coefficients are rational numbers. Tasks do not require any specific method to be used (e.g., prompts do not direct the student to use elimination or any other particular method). 	<p>MP.1 MP.7</p>
<p>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.F.BF.A.2,F.LE.A.2,F.LE.B.5</p>	<p>Concepts:</p> <ul style="list-style-type: none"> Recursion <p>Students are able to:</p> <ul style="list-style-type: none"> Distinguish between recursive and explicit formulas. Represent geometric and arithmetic sequences recursively. Represent geometric and arithmetic sequences with explicit formulas. Translate between recursive form and explicit form of geometric and arithmetic sequences. Recognize explicit formula for geometric sequences as 	<p>i</p>	<p>MP.1 MP.2 MP.4 MP.6 MP.7 MP.8</p>

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	<p>exponential functions containing a domain in the integers only.</p> <ul style="list-style-type: none"> • Interpret the parameters of an exponential function representing a geometric sequence. • Interpret the parameters of a linear function representing an arithmetic sequence. 		
<p>Use the formula for the sum of a finite geometric series to solve problems [<i>for example, calculate mortgage payments</i>; derive the formula for the sum of a finite geometric series (when the common ratio is not 1)].A.SSE.B.4</p>	<p>Concepts:</p> <ul style="list-style-type: none"> • Series as a sum of a sequence <p>Students are able to:</p> <ul style="list-style-type: none"> • Derive or explain the derivation of the formula for the sum of a finite geometric series. • Use the formula for the sum of a finite geometric series to solve problems. 	<ul style="list-style-type: none"> • In a multistep task, students may be expected to calculate the value of a single term as well as the sum. 	
<p>Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.N.RN.A.1, N,RN.A.2</p>	<p>Concepts:</p> <ul style="list-style-type: none"> • Properties of integer exponents extends to rational exponents (<i>for example, we define $5^{1/3}$ to be the cube root</i> 		MP.7

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	<p><i>of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5)</i></p> <ul style="list-style-type: none"> • Radical notation is a representation of rational exponents. <p>Students are able to:</p> <ul style="list-style-type: none"> • Rewrite expressions containing rational exponents into radical form. • Rewrite expressions containing radical notation into exponential expressions containing rational exponents. 		
<p>Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function. A.SSE.B.3, F.IF.C.8b</p>	<p>Concepts:</p> <ul style="list-style-type: none"> • Alternate, equivalent forms of an exponential expression containing rational exponents may reveal specific attributes of the function that it defines. <p>Students are able to:</p> <ul style="list-style-type: none"> • Use properties of exponent transform/rewrite an exponential expression for an exponential function. • Explain the properties of the quantity or the function. 	<ul style="list-style-type: none"> • Tasks have a real-world context. The equivalent form must reveal something about the real-world context. • Tasks require students to make the connection between the equivalent forms of the expression. 	<p>MP.1 MP.2 MP.4 MP.7</p>

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PPS Secondary Math Curriculum Algebra 2

<p>Express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.F.LE.A.4</p>	<p>Concepts:</p> <ul style="list-style-type: none"> • Exponents and logarithms have an inverse relationship. • Solutions to an exponential equation in one variable can be written as a logarithm. <p>Students are able to:</p> <ul style="list-style-type: none"> • Transform an exponential model represented by $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e. • Write the solution to $ab^{ct} = d$ as a logarithm. • Use technology to evaluate logarithms having base 2, 10, or e. 	<p>F-LE.A, Construct and compare linear, quadratic, and exponential models and solve problems, is the primary content and at least one of the other listed content elements will be involved in tasks as well.</p>	<p>MP.2 MP.4</p>
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Unit Vocabulary Terms

Unit Vocabulary Terms	
Index of a radical	Logarithm of y and with a base b
Simplest form of a radical	Natural logarithms
Power function	Exponential equation
Inverse relation	Logarithmic equation
Inverse function	
Radical function	
Radical equation	
Rational exponent	
Exponent	
Exponential function	
Exponential growth	
Growth factor	
Geometric Sequence	
Arithmetic Sequence	
Asymptote	
Exponential decay	
Decay factor	
Natural base e	

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Differentiations / Modifications Teaching Strategies

Research Based Effective Teaching Strategies	Modifications (how do I differentiate instruction?)	Special Education	Strategies for English Language Learners
<p>Task /Activities that solidifies mathematical concepts Use questioning techniques to facilitate learning</p> <p>Reinforcing Effort, Providing Recognition Practice, reinforce and connect to other ideas within mathematics</p> <p>Promotes linguistic and nonlinguistic representations</p> <p>Cooperative Learning Setting Objectives, Providing Feedback</p> <p>Varied opportunities for students to communicate mathematically</p> <p>Use technological and /or physical tools</p>	<p>Modifications Before or after school tutorial program Leveled rubrics Increased intervention Small groups Change in pace Calculators Extended time Alternative assessments Tiered activities/products Color coded notes Use of movements Use any form of technology</p> <p>*** Write the vocabulary using diagrams that indicate the meaning of the word. Have students explain the word</p> <p>*** construct, complete and use a table to help students organize variation functions. Table must</p>	<p>Change in pace Calculators Alternative assessments Accommodations as per IEP Modifications as per IEP Use graphic organizer to clarify mathematical functions for students with processing and organizing difficulties’.</p> <p>Constant review of math concepts to strengthen understanding of prior concepts for difficulties recalling facts.</p> <p>Use self-regulations strategies for student to monitor and assess their thinking and performance for difficulty attending to task</p> <p>Cooperative learning (small group, teaming, peer assisted tutoring) to foster communication and strengthen confidence.</p> <p>Use technology and/or hands on</p>	<p><u>Whiteboards</u> <u>Small Group / Triads</u> <u>Word Walls</u> <u>Partially Completed Solution</u> <u>Gestures</u> <u>Native Language Supports</u> <u>Pictures / Photos</u> <u>Partner Work</u> <u>Work Banks</u> <u>Teacher Modeling</u> <u>Math Journals</u> <u>Manipulatives</u> <u>Peer Coach</u></p> <p><i>Write the vocabulary using diagrams that indicate the meaning of the word. Have students explain the word</i></p> <p><i>While solving for rational expressions, students write or tell the procedures, demonstrating</i></p>

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<p>21st Century Learning Skills:</p> <p>Teamwork and Collaboration</p> <p>Initiative and Leadership</p> <p>Curiosity and Imagination</p> <p>Innovation and Creativity</p> <p>Critical thinking and Problem Solving</p> <p>Flexibility and Adaptability</p> <p>Effective Oral and Written Communication</p> <p>Accessing and Analyzing Information</p>	<p>include variation functions, corresponding equations and graphs</p> <p>Extension: <i>Discuss how to determine</i></p> <p>$f(x) = 5x + 105x + 10$ and $g(x) = \frac{11}{55}x - 2$ are inverses, challenge students to find to functions f and g such that $f(g(x)) \neq g(f(x))$ $(g(x)) \neq g(f(x))$, with one of the two compositions having the value x. Allow students to work in group to brainstorm solution to puzzle.</p> <p><i>Challenge students to explore whether is it possible to obtain extraneous solutions when solving an equation by cubing both side, as it is when squaring. Then have students extend their finding to raising both sides to any odd or even power. Ask student to share their solutions.</i></p>	<p>devices to: clarify abstract concepts and process for:</p> <ol style="list-style-type: none"> 1. Difficulty interpreting pictures and diagram. 2. difficulties with oral communications 3. Difficulty correctly identifying symbols of numeral 4. Difficulty maintaining attentions <p>Simplify and reduces strategies / Goal structure to enhance motivation, foster independence and self-direction for:</p> <ol style="list-style-type: none"> 1. difficulty attending to task 2. difficulty with following a sequence of steps to solution. 3. difficulty processing and organizing <p>Scaffolding math idea/concepts by guided practice and questioning strategies' to clarify and enhance understanding of math big ideas for:</p> <ol style="list-style-type: none"> 1. Difficulty with process and organization 2. difficulty with oral and written communication <p>Teacher models strategies' and think out aloud strategies to specify step by step process for:</p> <ol style="list-style-type: none"> 1. Difficulties processing and organization 2. Difficulty attending to tasks. <p>Use bold numbers and/or words to draw students' attention to important</p>	<p><i>with expressions</i></p> <p><i>Explain to students that extraneous solutions are "extra" as their name suggest.</i></p>
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		<p>information.</p> <p>Provide students with friendly numbers in order to focus on the mathematical concept rather than operations of the problem.</p> <p>***Students draw and cut rectangles. Then use the formula $d = \sqrt{l^2 + w^2}$ to find the length of each diagonal</p> <p>**While the algebraic method of verify that two functions are inverse functions is the most accurate method, it may not be helpful to solidify the concept for some students. Using technological resources, have students graph $f(x)$ and the line $y=x$ on the same coordinate plane. The inverse of f by reflecting its graph over the line $y=x$, and verify that the image is the graph of f^{-1}</p>	
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Instructional Resources and Materials

Instructional Resources and Materials	
Formative Assessment	Print
Short constructed responses Extended responses Checks for Understanding Exit tickets Teacher observation Projects Timed Practice Test – Multiple Choice & Open-Ended Questions <u>Performance Task:</u> <u><i>ILLUSTRATIVE MATHEMATICS PERFORMANCE" Forms of exponential expressions"</i></u> <u><i>ILLUSTRATIVE MATHEMATICS PERFORMANCE TASK "Carbon 14 dating"</i></u> <u>Performance Task for Class Use:</u>	McDougal Littell Algebra 2 (2007) <ul style="list-style-type: none"> • Chapter 3 Rational Exponents and Radical Functions • Chapter 4 Exponential and Logarithmic
	Technology
	Resources for teachers <u><i>Annenberg Learning : Insight into Algebra 1 Mathematics Assessment Projects</i></u> <u><i>Get the Math</i></u> <u><i>Achieve the Core</i></u> <u><i>Illustrative Mathematics</i></u> <u><i>Inside Mathmatics.org</i></u> <u><i>Asia Pacific Economic Cooperation : :Lesson Study Videos</i></u> <u><i>Genderchip.org</i></u> <u><i>Interactive Geometry</i></u> <u><i>Mathematical Association of America</i></u> <u><i>National Council of Teachers of Mathematics learner.org</i></u> <u><i>Math Forum : Teacher Place</i></u> <u><i>Shmoop /common core math</i></u> <u><i>Geometer's Sketchpad</i></u>

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PPS Secondary Math Curriculum Algebra 2

<p><u><i>N.CN.A.1 Complex number patterns</i></u> <u><i>N.CN.A.2 Powers of a complex number</i></u> <u><i>N.CN.C.7, A.REI.B.4b Completing the square</i></u> <u><i>A.REI.C.7 Linear and Quadratic System</i></u> <u><i>A.REI.C.6 Pairs of Whole Numbers</i></u> <u><i>F.BF.A.2 Snake on a Plane</i></u> <u><i>F.LE.A.2 Rumors</i></u> <u><i>F.LE.B.5, F.LE.A.2 Exponential Parameters</i></u> <u><i>A.SSE.B.4 Course of Antibiotics</i></u> <u><i>N.RN.A.1 Evaluating Exponential Expressions</i></u> <u><i>N.RN.A.2 Rational or Irrational?</i></u> <u><i>F.IF.C.8b Carbon 14 dating in practice I</i></u> <u>Project (optional)</u> <u><i>TEACH 21 PROBLEM BASED LEARNING PROJECT 'FATEFUL DECISION (GLOBAL WARMING)'</i></u></p>		
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