



**Plainfield Public Schools
Mathematics
Unit Planning Organizer**

| | |
|----------------------|--|
| Grade/Course | Algebra II |
| Unit of Study | Unit I Polynomials |
| Pacing | 8 weeks , 2 weeks for enrichment and re teaching |
| Dates | September 13 – November 2,2016 |

| <u>Standards for Mathematical Practices</u> |
|---|
| MP1. Make sense of problems and persevere in solving them. |
| MP2. Reason abstractly and quantitatively. |
| MP3. Construct viable arguments and critique the reasoning of others. |
| MP4. Model with mathematics. |
| MP5. Use appropriate tools strategically. |
| MP6. Attend to precision. |
| MP7. Look for and make use of structure. |
| MP8. Look for and express regularity in repeated reasoning. |

UNIT STANDARDS for Algebra II

HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4x^4 - y^4y^4$ as $(x^2x^2)^2 - (y^2y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2x^2 - y^2y^2)(x^2x^2 + y^2y^2)$.*

HSA.APR.B.3A. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

HSA.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

HSA.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

HSA.REI.D.11 Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic function

HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *

HSF.IF.C.7.C

Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior

HSA.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

HSA.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Additional Standards:

HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. *For example, **the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.***

HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix.

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| FOCUS STANDARD | | |
| HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ | | |
| Know | Remainder Theorem | 1 |
| Apply | | 2 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| FOCUS STANDARD | | |
| HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4x^4 - y^4y^4$ as $(x^2x^2)^2 - (y^2y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2x^2 - y^2y^2)(x^2x^2 + y^2y^2)$.</i> | | |
| Use | expression | 2 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| FOCUS STANDARD | | |
| HSA.APR.B.3A. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | |
| Identify | polynomial | 1 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| FOCUS STANDARD HSA.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | |
| Explain Construct | Equation Argument | 1 3 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| FOCUS STANDARD HSA.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | | |
| Solve | Rational equation Radical equations | 2 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| FOCUS STANDARD HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. * | | |
| Interpret | Graphs Tables | 3 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| FOCUS STANDARD HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | |
| Calculate Interpret | Rate of change | 2 3 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| FOCUS STANDARD HSA.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. | | |
| Rewrite | Rational expression | 2 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| SUPPORTING STANDARD HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior | | |
| Graph | polynomial | 2 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| FOCUS STANDARD HSA.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | | |
| Create Solve | Equations Inequalities | 4 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|---|---|------------|
| ADDITIONAL STANDARD HSA.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. <i>For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</i> | | |
| Prove | Polynomial | 4 |

| “Unwrapped” Skills (students need to be able to do) | “Unwrapped” Concepts (students need to know) | DOK Levels |
|--|---|------------|
| ADDITIONAL STANDARD HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix. | | |
| Derive | Parabola | 4 |

II. Standards Mathematical Practices..... Examples and Explanations

| <u>Standards</u> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
|---|---|---|
| <p>HS.A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.</p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> | <p>The Remainder theorem says that if a polynomial $p(x)$ is divided by $x - a$, then the remainder is the constant $p(a)$. That is, $p(x) = q(x)(x - a) + p(a)$. So if $p(a) = 0$ then $p(x) = q(x)(x - a)$.</p> <p>Example:</p> <ul style="list-style-type: none"> Let $p(x) = x^5 - 3x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does your answer tell you about the factors of $p(x)$? [Answer: $p(-2) = 0$ so $x + 2$ is a factor.] |
| <p>HS.A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> | <p>Graphing calculators or programs can be used to generate graphs of polynomial functions. Example:</p> <ul style="list-style-type: none"> Factor the expression $x^3 + 4x^2 - 59x - 126$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 59x - 126 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 - 59x - 126$. |
| <p>HS.A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships. <i>For example, differences of two squares ; the sum and differences of two cubes , the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</i></p> | <p><i>HS.MP.7.</i> Look for and make use of structure.</p> <p><i>HS.MP.8.</i> Look for and express regularity in repeated reasoning.</p> | <p>Examples:</p> <p>Use the distributive law to explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y.</p> <p>Derive the identity $(x - y)^2 = x^2 - 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by $-y$.</p> <p>Use an identity to explain the pattern</p> $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ |

Hyperlinks are noted underlined in italics

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| | | |
|---|---|--|
| <p>HS.A-APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> | <p><i>HS.MP.2. Reason abstractly and quantitatively.</i></p> <p><i>HS.MP.7. Look for and make use of structure.</i></p> | <p>[Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.]</p> <p>The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.</p> <p>Examples:</p> <ul style="list-style-type: none"> Find the quotient and remainder for the rational expression $\frac{x^3 - 3x^2 + x - 6}{x^2 + 2}$ and use them to write the expression in a different form. Express $f(x) = \frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. <p>[Answer: $f(x) = \frac{2x+1}{x-1} = \frac{2(x-1)+3}{x-1} = 2 + \frac{3}{x-1}$, so the horizontal asymptote is $y = 2$.]</p> |
| <p>HS.A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p> | <p>HS.MP.2. Reason abstractly and quantitatively.</p> <p>HS.MP.7. Look for and make use of structure.</p> | <p>Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further.</p> <p>Example:</p> <ul style="list-style-type: none"> Factor $x^3 - 2x^2 - 35x$ |
| | | |

Hyperlinks are noted underlined in italics

| Algebra: Reasoning with Equations and Inequalities ★ (A-REI) | | |
|---|---|---|
| Understand solving equations as a process of reasoning and explain the reasoning. | | |
| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
| <p>HS.A-REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>HS.MP.7.</i> Look for and make use of structure.</p> | <p>Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning. |
| <p>HS.A-REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.3.</i> Construct viable arguments and critique the reasoning of others.</p> <p><i>HS.MP.7.</i> Look for and make use of structure.</p> | <p>Examples:</p> <ul style="list-style-type: none"> • $\sqrt{x + 2} = 5$ • $\frac{7}{8}\sqrt{2x - 5} = 21$ • $\frac{x+2}{x+3} = 2$ • $\sqrt{3x - 7} = -4$ |

| Algebra: Reasoning with Equations and Inequalities ★ (A-REI) | | |
|--|---|--|
| Represent and solve equations and inequalities graphically. | | |
| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
| <p>HS.A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions</p> | <p>HS.MP.2. Reason abstractly and quantitatively.</p> <p>HS.MP.4. Model with mathematics.</p> <p>HS.MP.5. Use appropriate tools strategically.</p> <p>HS.MP.6. Attend to precision.</p> | <p>Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.</p> <p>Example:</p> <ul style="list-style-type: none"> Given the following equations determine the x value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x + 3)^2 - 1$ |

Algebra: Creating Equations ★ (A-CED)

Create equations that describe numbers or relationships.

| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
|--|---|--|
| <p>HS.A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> | <p>HS.MP.2. Reason abstractly and quantitatively.</p> <p>HS.MP.4. Model with mathematics.</p> <p>HS.MP.5. Use appropriate tools strategically.</p> | <p>Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.</p> <p>Examples:</p> <ul style="list-style-type: none"> Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation. <div data-bbox="1375 609 1507 701" style="text-align: center;"> </div> <ul style="list-style-type: none"> Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of a piece of lava t seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet? |

Functions: Interpreting Functions (F-IF)

Understand the concept of a function and use of function notation.

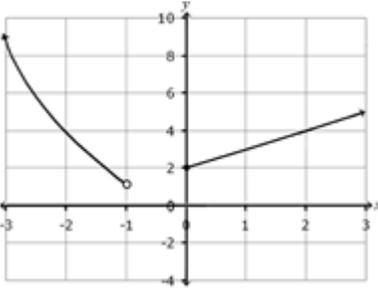
| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
|--|---|--|
| <p>HS.F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> <p><i>HS.MP.6.</i> Attend to precision.</p> | <p>Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.</p> <p>Examples:</p> <ul style="list-style-type: none"> • A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. <ul style="list-style-type: none"> ○ What is a reasonable domain restriction for t in this context? ○ Determine the height of the rocket two seconds after it was launched. ○ Determine the maximum height obtained by the rocket. ○ Determine the time when the rocket is 100 feet above the ground. ○ Determine the time at which the rocket hits the ground. ○ How would you refine your answer to the first question based on your response to the second and fifth questions? • Compare the graphs of $y = 3x^2$ and $y = 3x^3$. • Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. • Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. • It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday. |

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| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|--|-----|--------|----|---|----|----|---|----|---|-----|--|--------------|--------------|-----------------------|-----------------------|-----------------------|----|-------|-------|----|-------|-------|----|-------|-------|----|-------|-------|----|----|-------|--|--|--|
| <p>HS.F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> | <p><i>HS.MP.2.</i> Reason abstractly and quantitatively.</p> <p><i>HS.MP.4.</i> Model with mathematics.</p> <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> | <p>The average rate of change of a function $y = f(x)$ over an interval $[a,b]$ is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$</p> <p>In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Use the following table to find the average rate of change of g over the intervals $[-2, -1]$ and $[0,2]$: <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>$g(x)$</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>2</td> </tr> <tr> <td>-1</td> <td>-1</td> </tr> <tr> <td>0</td> <td>-4</td> </tr> <tr> <td>2</td> <td>-10</td> </tr> </tbody> </table> • The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track. <ul style="list-style-type: none"> ○ For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1. ○ How does the velocity of car 1 compare to that of car 2? <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Car 1</th> <th>Car 2</th> </tr> <tr> <th>d</th> <th>t</th> <th>t</th> </tr> </thead> <tbody> <tr> <td>10</td> <td>4.472</td> <td>1.742</td> </tr> <tr> <td>20</td> <td>6.325</td> <td>2.899</td> </tr> <tr> <td>30</td> <td>7.746</td> <td>3.831</td> </tr> <tr> <td>40</td> <td>8.944</td> <td>4.633</td> </tr> <tr> <td>50</td> <td>10</td> <td>5.348</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> | x | $g(x)$ | -2 | 2 | -1 | -1 | 0 | -4 | 2 | -10 | | Car 1 | Car 2 | d | t | t | 10 | 4.472 | 1.742 | 20 | 6.325 | 2.899 | 30 | 7.746 | 3.831 | 40 | 8.944 | 4.633 | 50 | 10 | 5.348 | | | |
| x | $g(x)$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| -2 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| -1 | -1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | -4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | -10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | Car 1 | Car 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| d | t | t | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 4.472 | 1.742 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 20 | 6.325 | 2.899 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 30 | 7.746 | 3.831 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 40 | 8.944 | 4.633 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 50 | 10 | 5.348 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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Hyperlinks are noted underlined in italics

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| <u>Standards</u> <i>Students are expected to:</i> | <u>Mathematical Practices</u> | <u>Explanations and Examples</u> |
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| <p>HS.F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> | <p><i>HS.MP.5.</i> Use appropriate tools strategically.</p> <p><i>HS.MP.6.</i> Attend to precision.</p> | <p>Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.</p> <p>Examples:</p> <ul style="list-style-type: none"> Describe key characteristics of the graph of $f(x) = x - 3 + 5$. Sketch the graph and identify the key characteristics of the function described below. |
| <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> | | $F(x) = \begin{cases} x + 2 & \text{for } x \geq 0 \\ -x^2 & \text{for } x < -1 \end{cases}$  <ul style="list-style-type: none"> Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs? |

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Essential QuestionsCorresponding Big Ideas

| Essential Questions | Corresponding Big Ideas |
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| How is multiplying any two polynomials just an expansion of the distributive property? | Functions apply to a wide range of situations. They do not have to be described in any specific expression or follow a regular pattern. They apply to cases other than those of “continuous variation”.For example, sequences are functions. |
| How do you compose two functions to make a new function? | For functions that map real numbers to real numbers , certain patterns of covariation, or patterns in how two variables changes together , indicate membership in a particular family of functions and determine the type of formula that has the function has. |
| How do you factor differences and sums of cubes? | A rate of change describes how one variables quantity changes with respect to another -in other words, a rat of change describes the covariation between two variables. |
| How do I find the inverse of a function? | A function’s rate change is one of the main characteristic that determine what kinds of real whole phenomena the function can model. |
| How do I factor sum and difference of cubes? | Quadratic functions are characterized by a linear rate of change of the rate change (the second derivative) of a quadratic function is constant. Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic are real, they are symmetric about the x -coordinate of the maximum or minimum point. |
| How can solving polynomial equations be useful when graphing polynomial functions? | Functions that have the same domain and that map to the real numbers can be added, subtracted, multiplied or divided (which may change the domain). |
| Why is a zero important for a polynomial? | For functions that map the real numbers to the real numbers, composing a functions with “shifting” or scaling” functions changes the formula and graph of the functions in readily predictable ways. |
| How do I use the quadratic formula to solve quadratic equations? | |

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| | <p>Under appropriate conditions, functions have inverses</p> <p>Functions can be represented in various ways, including through algebraic means (e.g, equation), graphs, word descriptions, and tables .</p> <p>Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristic, and some may show only part of the function.</p> <p>Some representations of a function may be more useful than others, depending on the context.</p> <p>Links between algebraic and graphical representations of a functions are especially important in the studying relationship and change.</p> <p>Sources: Cooney, T & Beckman, Sybilla. (2010). Developing essential understanding of Functions 9-12. Reston, VA: The National Council of Teachers of Mathematics, Inc.</p> |
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Student Learning Objectives

| Student Learning Objective | Skill /Concepts | <u>PARCC Evidence Table. Math Test Specifications</u> | Mathematical Practices |
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| <p>Apply the Remainder Theorem in order to determine the factors of a polynomial. A.APR.B.2</p> | <p>Concepts:</p> <ul style="list-style-type: none"> • Polynomial division: For a polynomial $p(x)$ and a number a: <ul style="list-style-type: none"> – $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ – $(x - a)$ is a factor of $p(x)$ if and only if $p(a) = 0$ <p>Students are able to:</p> <ul style="list-style-type: none"> • use the Remainder Theorem to determine factors of a polynomial. | | MP.6 |
| <p>Use an appropriate factoring technique to factor polynomials. Explain the relationship between zeros and factors of polynomials, and use the zeros to construct a rough graph of the function defined by the polynomial. A.SSE.A.2, A.APR.B.3</p> | <p>Concepts:</p> <p>Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function.</p> <p>Students are able to:</p> <ul style="list-style-type: none"> • factor polynomials. • analyze a table of values to determine where the polynomial is increasing and decreasing. • use the zeros of the polynomial to create rough graph. | <ul style="list-style-type: none"> ▪ Tasks will not include sums and differences of cubes | MP 7 |
| <p>Graph polynomial functions from equations; identify zeros when suitable factorizations are available;</p> | <p>Concepts:</p> <ul style="list-style-type: none"> • Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function. | <ul style="list-style-type: none"> ▪ Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated | MP.1, MP.2, MP.3, MP.6, MP7 |

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| <p>show key features and end behavior. F.IF.C.7, F.IF.C.7c</p> | <p>Students are able to:</p> <ul style="list-style-type: none"> graph a polynomial function given its equation. identify zeros from the graph and using an appropriate factoring technique. show key features of the graph, including end behavior. use technology to graph and describe key features of the graph for complicated cases. | <p>across: Tasks will draw on securely held content from previous grades and courses, including down to Grade 7, but that are at the Algebra II level of rigor.</p> <ul style="list-style-type: none"> Tasks will synthesize multiple aspects of the content listed in the evidence statement text, but need not be comprehensive. Tasks should address at least A-SSE.A.1b, A-REI.A.1, and F-IF.A.2 and either FIF.C.7a or F-IF.C.7e (excluding trigonometric and logarithmic functions). | |
| <p>Use polynomial identities to describe numerical relationships and prove polynomial identities. A.APR.C.4</p> | <p>Concepts:</p> <ul style="list-style-type: none"> Polynomial identities can be used to describe numerical relationships. <p>Students are able to:</p> <ul style="list-style-type: none"> show that the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. prove polynomial identities. | <ul style="list-style-type: none"> Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures | <p>MP.3</p> |
| <p>Rewrite simple rational expressions in different forms using inspection A.APR.D.6,</p> | <p>Concepts:</p> <ul style="list-style-type: none"> Rational expressions can be written in different forms. <p>Students are able to:</p> <ul style="list-style-type: none"> write $a(x)/b(x)$ in the form $q(x) +$ | <ul style="list-style-type: none"> Examples will be simple enough to allow inspection or long division. Simple rational expressions are limited to | <p>MP.1</p> |

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| | <p>$r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.</p> <ul style="list-style-type: none"> • use inspection, factoring and long division to rewrite rational expressions. • use technology to rewrite rational expressions for more complicated cases. | <p>numerators and denominators that have degree at most 2.</p> | |
| <p>Solve simple rational and radical equations in one variable, use them to solve problems and show how extraneous solutions may arise. Create simple rational equations in one variable and use them to solve problems. A.REI.A.2, A.REI.A.1, A.CED.A.</p> | <p>Concepts: Inverse relationships exist between roots and powers. Extraneous solutions do not result in true statements.</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ▪ •use the inverse relationship between roots and powers when solving radical equations. ▪ •identify any extraneous solutions. ▪ •solve simple rational equations in one variable (degree of numerators and denominator is not greater than 2). ▪ •write simple rational equations in one variable and use the rational equation to solve problems. | <ul style="list-style-type: none"> ▪ Simple rational equations are limited to numerators and denominators that have degree at most 2. ▪ Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to simple rational or radical equations. | <p>MP.3 MP.6</p> |

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| <p>For radical functions, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.B.4., F.IF.B.6</p> | <p>Concepts:</p> <ul style="list-style-type: none"> • A radical function is any function that contains a variable inside a root. <p>Students are able to:</p> <ul style="list-style-type: none"> • interpret key features of radical functions from graphs and tables in the context of the problem. • sketch graphs of radical functions given a verbal description of the relationship between the quantities. • identify intercepts and intervals where function is increasing/decreasing. • determine the practical domain of a radical function. • determine key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior. | <ul style="list-style-type: none"> ▪ Key features may also include discontinuities ▪ Tasks have a real-world context. Tasks must include the interpret part of the evidence statement. ▪ Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. | <p>MP.1, MP.4, MP.5, MP.7</p> |
| <p>Graph logarithmic functions expressed symbolically and show key features of the graph (including intercepts and end behavior). F.IF.C.7, F.IF.C.7e</p> | <p>Concepts: Logarithmic functions</p> <p>Students are able to:</p> <ul style="list-style-type: none"> • graph logarithmic functions having base 2, 10 or e, using technology for more complicated cases. • show intercepts and end behavior of logarithmic functions. | <ul style="list-style-type: none"> ▪ Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: Tasks will draw on securely held content from previous | <p>MP.1, MP.2, MP.3, MP.6, MP.7</p> |

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| | | <p>grades and courses, including down to Grade 7, but that are at the Algebra II level of rigor.</p> <ul style="list-style-type: none">▪ Tasks will synthesize multiple aspects of the content listed in the evidence statement text, but need not be comprehensive.▪ Tasks should address at least A-SSE.A.1b, A-REI.A.1, and F-IF.A.2 and either FIF.C.7a or F-IF.C.7e (excluding trigonometric and logarithmic functions).▪ Tasks should also draw upon additional content listed for grades 7 and 8 and from the remaining standards in the Evidence Statement Text. 7 | |
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| <p>Find approximate solutions for $f(x)=g(x)$, using technology to graph, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, logarithmic and exponential functions. A.REI.D.11.</p> | <p>Concepts:</p> <ul style="list-style-type: none"> Solutions to complex systems of nonlinear functions can be approximated graphically <p>Students are able to:</p> <ul style="list-style-type: none"> find the solution to $f(x)=g(x)$ approximately, e.g., using technology to graph the functions; include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. find the solution to $f(x)=g(x)$ approximately, e.g., using technology to make tables of values, or find successive approximations; include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | <ul style="list-style-type: none"> Given an equation or system of equations, reason about the number or nature of the solutions <ul style="list-style-type: none"> For example, students might be asked how many positive solutions there are to the equation $ex = x+2$ or the equation $ex = x+1$, explaining how they know. The student might use technology strategically to plot both sides of the equation without prompting | <p>MP 3</p> |
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Unit Vocabulary Terms

| Unit Vocabulary Terms | |
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| <p>Zeros of polynomials, Geometric series finite series complex solutions. Complex number $(a+bi)$ Relative maximum Relative minimum Symmetries End behavior The Factor Theorem Quadratic formula Difference of sums Sums of cubes</p> | <p>Rational expression Irrational numbers Irrational number Fundamental Theorem of Algebra The Remainder Theorem Conjugate Factorization finite, square roots, real numbers. Natural numbers Monomials base, exponents, coefficients degree power of polynomials. complex numbers.</p> |

Differentiations / Modifications Teaching Strategies

| Research Based Effective Teaching Strategies | Modifications (How do I differentiate instruction?) | Special Education | Strategies for English Language Learners |
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| <p>Task /Activities that solidifies mathematical concepts Use questioning techniques to facilitate learning</p> <p>Reinforcing Effort, Providing Recognition Practice, reinforce and connect to other ideas within mathematics</p> <p>Promotes linguistic and nonlinguistic representations</p> <p>Cooperative Learning Setting Objectives, Providing Feedback</p> <p>Varied opportunities for students to communicate mathematically</p> <p>Use technological and /or physical tools</p> | <p>Modifications Before or after school tutorial program Leveled rubrics Increased intervention Small groups Change in pace Calculators Extended time Alternative assessments Tiered activities/products Color coded notes Use of movements Use any form of technology</p> <p>Students work in cooperative groups writes a polynomial function. Each group decides if the function is an even degree or odd degree function. Students predict the end behavior ,and the number of zeros , student</p> | <p>Change in pace Calculators Alternative assessments Accommodations as per IEP Modifications as per IEP Use graphic organizer to clarify mathematical functions for students with processing and organizing difficulties’.</p> <p>Constant review of math concepts to strengthen understanding of prior concepts for difficulties recalling facts.</p> <p>Use self-regulations strategies’ for student to monitor and assess their thinking and performance for difficultly attending to task</p> <p>Cooperative learning (small group, teaming, peer assisted tutoring) to foster communication and strengthen confidence.</p> | <p><u>Whiteboards</u> <u>Small Group / Triads</u> <u>Word Walls</u> <u>Partially Completed Solution</u> <u>Gestures</u> <u>Native Language Supports</u> <u>Pictures / Photos</u> <u>Partner Work</u> <u>Work Banks</u> <u>Teacher Modeling</u> <u>Math Journals</u> <u>Manipulatives</u> <u>Peer Coach</u></p> <p>Students explain how to find all rational zeros of a polynomial function. Students model and articulate techniques of polynomial functions of degree 3 or higher</p> |

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| <p>21st Century Learning Skills :</p> <p>Teamwork and Collaboration</p> <p>Initiative and Leadership</p> <p>Curiosity and Imagination</p> <p>Innovation and Creativity</p> <p>Critical thinking and Problem Solving</p> <p>Flexibility and Adaptability</p> <p>Effective Oral and Written Communication</p> <p>Accessing and Analyzing Information</p> | <p>check prediction using a calculator</p> <p>Students write explanation for: how to make a table for polynomial functions; how to plot a function; how to locate the number of zeros for each function</p> <p>Extension: Using concepts from unit students create error analysis. On one side of a card students write problems and incorrectly solve them. On opposite side of the card, students provide correct solutions. Students share card with peers.</p> | <p>Use technology and/or hands on devices to: clarify abstract concepts and process for:</p> <ol style="list-style-type: none"> 1. Difficulty interpreting pictures and diagram. 2. Difficulties with oral communications 3. Difficulty correctly identifying symbols of numeral 4. Difficulty maintaining attentions <p>Simplify and reduces strategies / Goal structure to enhance motivation, foster independence and self-direction for:</p> <ol style="list-style-type: none"> 1. difficulty attending to task 2. difficulty with following a sequence of steps to solution. 3. difficulty processing and organizing <p>Scaffolding math idea/concepts by guided practice and questioning strategies' to clarify and enhance understanding of math big ideas for:</p> <ol style="list-style-type: none"> 1. Difficulty with process and organization 2. Difficulty with oral and written communication <p>Teacher models strategies' and think out aloud strategies to specify step by step process for:</p> <ol style="list-style-type: none"> 1. Difficulties processing and organization 2. Difficulty attending to tasks. <p>Use bold numbers and/or words to draw students' attention to important information.</p> <p>Provide students with friendly</p> | |
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| | | <p>numbers in order to focus on the mathematical concept rather than operations of the problem.</p> <p>Have students note and explain the differences between the use of the symbol (-) for subtraction sentences and for identifying negative numbers. Help make connections between everyday situations and the use of positive numbers, negative numbers, or zero by making posters depicting different scenarios, example a lemonade stand. Include illustrations and paragraph explaining the situation. Show coins and paper money to illustrate the problem. Represent a loss with (-) and a profit or gain with (+). Then have students solve problems about the scenario.</p> | |
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Instructional Resources

| Instructional Resources and Materials | | | |
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| Formative Assessment | Print | | |
| <p>Short constructed responses Extended responses Checks for Understanding Exit tickets Teacher observation Projects Timed Practice Test – Multiple Choice & Open-Ended Questions</p> <p><u>Performance Task:</u></p> <p><u>A Cubic Identity aligned to A.SSE. A.2, A.SSE.B3</u></p> <p><u>An Extraneous Solution aligned to A.REI.A.2,A.CED.A.1</u></p> <p><u>Summative</u></p> <p>End of Unit Assessment for Algebra 2 Unit 1 Polynomials</p> | <p>McDougal Little Algebra 2 (2007)</p> <ul style="list-style-type: none"> ▪ Chapter 5 Polynomials and Polynomial Functions ▪ Chapter 6 Rational Exponents and Radical Functions ▪ Chapter 7 Exponential and Logarithm Functions <p style="background-color: #d3d3d3; text-align: center;">Technology</p> <table border="0" style="width: 100%;"> <tr> <td style="vertical-align: top; width: 50%;"> <p>Resources for teachers</p> <p><u>NJ CORE</u> <u>Mathematics Assessment Projects</u> <u>Get the Math</u> <u>Achieve the Core</u> <u>Webmath.com</u> <u>sosmath.com</u> <u>Mathplanet.com</u> <u>Interactive Mathematics.com</u> <u>Illustrative Mathematics</u> <u>Inside Mathmatics.org</u> <u>Asia Pacific Economic Cooperation : :Lesson Study Videos</u> <u>Genderchip.org</u> <u>Interactive Geometry</u> <u>Mathematical Association of America</u> <u>National Council of Teachers of Mathematics learner.org</u> <u>Math Forum : Teacher Place</u> <u>Shmoop /common core math</u></p> </td> <td style="vertical-align: top; width: 50%;"> <p>Resources for Students</p> <p><u>Khan Academy</u> <u>Math world : Wolfram.com</u> <u>Webmath.com</u> <u>sosmath.com</u> <u>Mathplanet.com</u> <u>Interactive Mathematics.com</u> <u>Mathexpression.com.algebra</u> <u>Math Words for Advance Algebra & Pre-Calculus</u> <u>Math TV</u></p> </td> </tr> </table> | <p>Resources for teachers</p> <p><u>NJ CORE</u> <u>Mathematics Assessment Projects</u> <u>Get the Math</u> <u>Achieve the Core</u> <u>Webmath.com</u> <u>sosmath.com</u> <u>Mathplanet.com</u> <u>Interactive Mathematics.com</u> <u>Illustrative Mathematics</u> <u>Inside Mathmatics.org</u> <u>Asia Pacific Economic Cooperation : :Lesson Study Videos</u> <u>Genderchip.org</u> <u>Interactive Geometry</u> <u>Mathematical Association of America</u> <u>National Council of Teachers of Mathematics learner.org</u> <u>Math Forum : Teacher Place</u> <u>Shmoop /common core math</u></p> | <p>Resources for Students</p> <p><u>Khan Academy</u> <u>Math world : Wolfram.com</u> <u>Webmath.com</u> <u>sosmath.com</u> <u>Mathplanet.com</u> <u>Interactive Mathematics.com</u> <u>Mathexpression.com.algebra</u> <u>Math Words for Advance Algebra & Pre-Calculus</u> <u>Math TV</u></p> |
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