Plainfield Public Schools
Mathematics
Unit Planning Organizer

Grade	Accelerated 7
Unit of Study	**Unit 2 Proportional Reasoning and Equations**
Pacing	7 weeks ; 5 instructional weeks 2 weeks for re-teaching/enrichment

Standard for Mathematical Practices
MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

I. Unit Standards

UNIT STANDARDS
7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ to ¼ miles per hour, equivalently 2 miles per hour. **7. RP.2. Recognize and represent proportional relationships between quantities.** a. **Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.** b. **Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.** c. **Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.** d. **Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.** **7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.** **7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10percent raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches***

Hyperlinks are noted underlined in italics

from each edge; this estimate can be used as a check on the exact computation.

7.EE. 4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a) **Solve word problems leading to equations of the form *px + q = r* and *p(x + q) = r,* where *p, q, and r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?***

b) **Solve word problems leading to equations of the form *px + q > r or px + q < r,* where *p, q, and r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week***

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed*

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

8.EE.C.7 Solve linear equations in one variable

8.EE.C.7.A Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers)

8.EE.C.7 Solve linear equations in one variable

8.EE.C.7.B Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms

Hyperlinks are noted underlined in italics

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
FOCUS STANDARD: **7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ to ¼ miles per hour, equivalently 2 miles per hour*.**		
Compute	Unit rate	3

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
FOCUS STANDARD: **7.RP.2. Recognize and represent proportional relationships between quantities.** a. **Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.** b. **Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.** c. **Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.** d. **Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.**		
Recognize Represent Decide Identify Explain	Proportional relationships Equivalent ratios Constant of proportionality Point (x,y) Multi-step problems	2 3 4 2 4

Hyperlinks are noted underlined in italics

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
Focus Standard **8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed***		
Graph	Proportional relationships	2

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
Focus Standard **8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.**		
Use	slope	2

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
FOCUS STANDARD: **7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.** *For example: If a woman making $25 an hour gets a 10 percent raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*		
Solve Apply	Rational numbers	2 2

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"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
FOCUS STANDARD: **7. EE.B. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.**		
Use	Variables	2

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
Focus Standard **8.EE.C.7 Solve linear equations in one variable** **8.EE.C.7.A Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers)**		
Give examples	Linear equation	2

"Unwrapped" Skills (students need to be able to do)	"Unwrapped" Concepts (students need to know)	DOK Levels
8.EE.C.7 Solve linear equations in one variable **Focus Standard** **8.EE.C.7.B Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms**		
Solve	Linear equations	2

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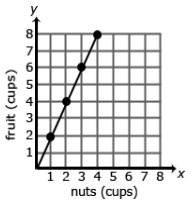
II. Mathematical Standards and Practice …. Explanation and Examples

<table>
<tr><th colspan="3">Ratios and Proportional Relationships (RP)
Analyze proportional relationships and use them to solve real-world and mathematical problems.</th></tr>
<tr><td>Standards
Students are expected to:</td><td>Mathematical Practices</td><td>Explanations and Examples</td></tr>
<tr><td>7.RP.A.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.</td><td>7.MP.2. Reason abstractly and quantitatively.
7.MP.6. Attend to precision.</td><td></td></tr>
<tr><td>7.RP.A.2. Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</td><td>7.MP.1. Make sense of problems and persevere in solving them.
7.MP.2. Reason abstractly and quantitatively.
7.MP.3. Construct viable arguments and critique the reasoning of others.
7.MP.4. Model with mathematics.
7.MP.5. Use appropriate tools strategically.
7.MP.6. Attend to precision.</td><td>Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.
Examples:
● A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.
<table><tr><th>Serving Size</th><th>1</th><th>2</th><th>3</th><th>4</th></tr><tr><td>Cups of Nuts (x)</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Cups of Fruit (y)</td><td>2</td><td>4</td><td>6</td><td>8</td></tr></table> 
Continued on the next page</td></tr>
</table>

<table>
<tr>
<td>7.RP.A.2. continued

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.</td>
<td>7.MP.7. Look for and make use of structure.

7.MP.8. Look for and express regularity in repeated reasoning.</td>
<td>The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).

The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.

● The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

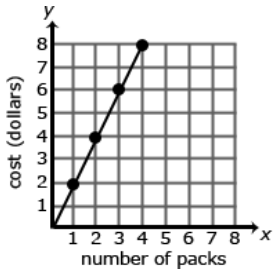
 

Table:

<table>
<tr><th>Number of Packs of Gum (g)</th><th>Cost in Dollars (d)</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>6</td></tr>
<tr><td>4</td><td>8</td></tr>
</table>

Equation: 2g = d, where d is the cost in dollars and g is the packs of gum

o A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost (g x 2 = d).</td>
</tr>
</table>

Hyperlinks are noted underlined in italics

<table>
<tr>
<td>7.RP.A.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</td>
<td>7.MP.1. Make sense of problems and persevere in solving them.

7.MP.2. Reason abstractly and quantitatively.

7.MP.3. Construct viable arguments and critique the reasoning of others.

7.MP.4. Model with mathematics.

7.MP.5. Use appropriate tools strategically.

7.MP.6. Attend to precision.

7.MP.7. Look for and make use of structure.

7.MP.8. Look for and express regularity in repeated reasoning.</td>
<td>Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

Examples:
● Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say: “The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40.”

$4.17 + 4.17 + (0.24 · 4.17) = 2.24 x 4.17

<table><tr><td>100%</td><td>100%</td><td>24%</td></tr><tr><td>$4.17</td><td>$4.17</td><td>?</td></tr></table>
● A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

<table><tr><td colspan="2">37.50</td></tr><tr><td>33% of 37.50</td><td>67% of 37.50</td></tr></table>
The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.</td>
</tr>
</table>

Hyperlinks are noted underlined in italics

7.RP.A.3. *continued*		• A shirt is on sale for 40% off. The sale price is \$12. What was the original price? What was the amount of the discount?  0.60p = 12 • At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution. • A salesperson set a goal to earn \$2,000 in May. He receives a base salary of \$500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal? • After eating at a restaurant, your bill before tax is \$52.60. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill. The amount paid = 0.20 x \$52.50 + 0.08 x \$52.50 = 0.28 x \$52.50.

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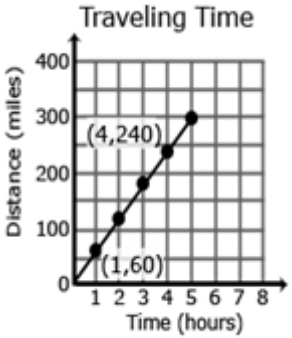
Expressions and Equations (EE)

Understand the connections between proportional relationships, lines, and linear equations.

Standards *Students are expected to:*	*Mathematical Practices*	*Explanations and Examples*
8.EE.B.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	*8.MP.1.* Make sense of problems and persevere in solving them. *8.MP.2.* Reason abstractly and quantitatively. *8.MP.3.* Construct viable arguments and critique the reasoning of others. *8.MP.4.* Model with mathematics. *8.MP.5.* Use appropriate tools strategically. *8.MP.6.* Attend to precision. *8.MP.7.* Look for and make use of structure. *8.MP.8.* Look for and express regularity in repeated reasoning.	Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs. Example: • Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation. Scenario 1  Scenario 2: •

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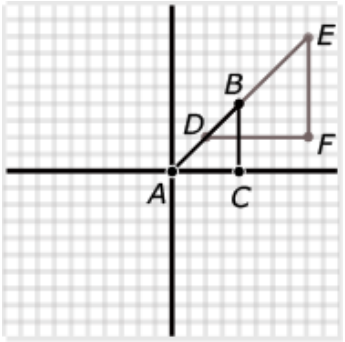
8.EE.B.6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.	8.*MP.2.* Reason abstractly and quantitatively. *8.MP.3.* Construct viable arguments and critique the reasoning of others. *8.MP.4.* Model with mathematics. *8.MP.5.* Use appropriate tools strategically. *8.MP.7.* Look for and make use of structure. *8.MP.8.* Look for and express regularity in repeated reasoning.	• Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation. 

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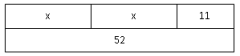
Expressions and Equations (EE) **Use properties of operations to generate equivalent expressions.**		
Standards *Students are expected to:*	*Mathematical Practices*	*Explanations and Examples*
7.EE.B.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	*7.MP.1.* Make sense of problems and persevere in solving them. *7.MP.2.* Reason abstractly and quantitatively. *7.MP.3.* Construct viable arguments and critique the reasoning of others. *7.MP.4.* Model with mathematics. *7.MP.5.* Use appropriate tools strategically. *7.MP.6.* Attend to precision. *7.MP.7.* Look for and make use of structure. *7.MP.8.* Look for and express regularity in repeated reasoning.	Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include, but are not limited to: ● front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), ● clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), ● rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), ● using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and ● using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). **Example**: ● The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.  2x + 11 = 52 2x = 41 x = $20.5
7.EE.B.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	*7.MP.1.* Make sense of problems and persevere in solving them. *7.MP.2.* Reason abstractly and quantitatively.	**Examples**: ● Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost? ● The sum of three consecutive even numbers is 48. What is the smallest of these

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a. Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px+q>r$ or $px+q<r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions*	*7.MP.3.* Construct viable arguments and critique the reasoning of others. *7.MP.4.* Model with mathematics. *7.MP.5.* Use appropriate tools strategically. *7.MP.6.* Attend to precision. *7.MP.7.* Look for and make use of structure. *7.MP.8.* Look for and express regularity in repeated reasoning.	numbers? • Solve: $\frac{5}{4}n+5=20$ • Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase. • Steven has $25 dollars. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy? Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution. • Solve $\frac{1}{2}x+3>2$ and graph your solution on a number line.

Expressions and Equations (EE)

Analyze and solve linear equations and pairs of simultaneous linear equations.

Standards *Students are expected to:*	Mathematical Practices	Explanations and Examples

Hyperlinks are noted underlined in italics

8.EE.C.7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	8.MP.2. Reason abstractly and quantitatively. 8.MP.5. Use appropriate tools strategically. 8.MP.6. Attend to precision. 8.MP.7. Look for and make use of structure.	As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions. When the equation has one solution, the variable has one value that makes the equation true as in 12 - 4*y* = 16. The only value for y that makes this equation true is -1. When the equation has infinitely many solutions, the equation is true for all real numbers as in 7*x* + 14 = 7 (*x*+2). As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution. When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in 5*x* - 2 = 5(*x*+1). When simplifying this equation, students will find that the solution appears to be two numbers that are not equal or -2 = 1. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution. Examples: ● Solve for x: o $-3(x+7)=4$ o $3x-8=4x-8$ o $3(x+1)-5=3x-2$ ● Solve: o $7(m-3)=7$ o $\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$

Hyperlinks are noted underlined in italics

III. Essential Questions …… Corresponding Big Ideas

Essential Questions	Corresponding Big Ideas
How does ratio reasoning differ from other types of reasoning? How are ratios related to fractions? How are proportions like a seesaw? How do I use rates to describe changes in our daily lives? How does writing a situation as an expression or equation simplify my life? How are expressions equivalent? How can patterns, and relationships be used as tools to best describe and help explain real-life situations? How can we solve real-world mathematical problems using numerical and algebraic expressions and equations? How do we solve inequalities and determine the solution set? How does writing a situation as an	Reasoning with ratios involves attending to and coordinating two quantities. A ratio is a multiplicative comparison of two quantities, or is a joining two quantities in composed unit. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest. A number of mathematical connections link ratios and fractions: Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. ● Ratios and fractions can be thought of as overlapping sets. ● Ratios can often be meaningfully reinterpreted as fractions. ● Ratios can be meaningfully reinterpreted as quotients. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change. Proportional reasoning is complex and involves understanding that - ● Equivalent ratios can be created by iterating and/or partitioning a composed unit; ● If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and ● The two types of ratios - composed units and multiplicative comparisons - are related. A rate is set of infinitely many equivalent ratios. Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.

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expression or equation simplify my life? How are expressions equivalent? How can patterns, and relationships be used as tools to best describe and help explain real-life situations? How can we solve real-world mathematical problems using numerical and algebraic expressions and equations? How do we solve inequalities and determine the solution set?	The equal signs can be used in defining or giving a name to an expression of function rule. Expressions are powerful tools for exploring, reasoning about, and representing situations. Two or more expressions may be equivalent, even when their symbolic forms differ. Using variables permits writing expressions whose values are not known or vary under different circumstances. It is often important to find the value(s) of a variable for which two expressions represent the same quantity Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving problems Finding the value(s) of a variable for which two expression(s) represent the same quantity is known as solving an equation An inequality is another way to describe a relationship between expressions; instead of showing that the values of two expressions are equal, inequalities indicate that the value of one expression is greater than (or greater than or equal to) the value of the other expression. *Source:* *Lloyd, G., Herbel-Eisenmann, B., & Star, J. R Developing essential understanding of expressions, equations, and functions for teaching mathematics in grades 6-8. Reston, VA: . (2011). The National Council of Teachers of Mathematics, Inc* *Lobato, J. E. (2010). Developing essential understanding of ratios, proportions & proportional reasoning for teaching mathematics in grades 6-8. Reston, VA: The National Council of Teachers of Mathematics, Inc.*

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IV. Unit Vocabulary Terms

Ratio Proportion Rate equal Cross Product Constant of Proportionality Scale Percent Percent of change Proportion Simple Interest Tax Percent error Percent of increase/decrease Gratuities Commissions Discount Markup Interest Markup Markdown	Dependent variable Independent variable Linear relationship Coefficient X intercept y-intercept equivalent expressions inequality point of intersection properties of equality slope function additive inverse multiplicative inverse

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V. Student Learning Objectives

Student Learning Objectives	Skills/Concept	***PARCC Instructional Clarification Mathematics Assessment Test Specifications***	Mathematical Practice
Calculate and interpret unit rates of various quantities involving ratios of fractions that contain like and different units.7.RP.1	**Students are able to:** • compute unit rates with ratios of fractions. • compute unit rates with ratios of fractions representing measurement quantities. in both like and different units of measure.	• Tasks have a real-world context. • Tasks do not assess unit conversions.	MP.2 MP.4 MP.6
Determine if a proportional relationship exists between two quantities e.g. by testing for equivalent ratios in a table or graph on the coordinate plane and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) from tables, graphs, equations, diagrams, and verbal descriptions. Write equations to model proportional relationships in real world. Use the graph of a proportional relationship to interpret the meaning of any point (x, y) on the	**Concept(s):** • Proportions represent equality between two ratios. • Constant of proportionality **Students are able to:** • use tables and graphs to determine if two quantities are in a proportional relationship. • identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. • write equations representing proportional relationships. • Interpret the origin and (1, r) on the graph of a	• Tasks have “thin context”1 or no context. • Tasks are not limited to ratios of whole numbers. • Tasks sample equally across the listed representations (graphs, equations, diagrams, and verbal descriptions). • Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality.	MP.2 MP.5

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graph in terms of the situation - including the points (0, 0) and (1, r), recognizing that r is the unit rate..RP.2	proportional relationship in context. • interpret a point on the graph of a proportional relationship in context.		
Graph proportional relationships, interpreting slope as unit rate, and compare two proportional relationships, each represented in different ways. 8.EE.B.5	**Concept(s):** • Quantitative relationships can be represented in different ways. **Students are able to:** • Graph proportional relationships. • Interpret unit rate as the slope of a graph. • Compare two different proportional relationships that are represented indifferent ways (table of values, equation, graph, verbal description).	▪ Tasks may or may not contain context.	MP.2 MP.4 MP.5 MP.6 MP.7 MP.8
Derive the equation of a line (y = mx for a line through the origin and the equation y = mx +b for a line intercepting the vertical axis at b) and use similar triangles to explain why the slope (m) is the same between any two points on a non-vertical line in the coordinate plane. 8.EE.B.6	**Students are able to:** • Show, using similar triangles, and explain why the slope, m, is the same between any two distinct points on a non-vertical line. • Derive, from two points, the equation y = mx for a line through the origin. • Derive, from two points, the equation y = mx + b for a line	▪ Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. ▪ Tasks require students to derive the equation y=mx for a line through the origin and the equation y=mx+b for a line intersecting the	MP.2 MP.4 MP.5 MP.6 MP.7 MP.8

	intercepting the vertical axis at b.	vertical axis at b. ▪ Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions.	
Solve multi-step ratio and percent problems using proportional relationships (*simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error)*	**Concept(s):** • Recognize percent as a ratio indicating the quantity *per one hundred*. **Students are able to:** • use proportions to solve multistep percent problems including simple interest, tax, markups, discounts, gratuities, commissions, fees, percent increase, percent decrease, percent error. • use proportions to solve multistep ratio problems.	● Tasks will include proportional relationships that only involve positive numbers. ● Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality	MP.1 MP2 MP.3 MP.6 MP.7 MP.8
Solve multi-step real life and mathematical problems with rational numbers in any form (fractions, decimals) by applying properties of operations and converting rational numbers between forms as needed. Assess the reasonableness of answers using mental computation and estimation strategies.7.EE.	**Concept(s):** ● Rational numbers can take different forms. **Students are able to:** • solve multi-step real-life problems using rational numbers in any form. • solve multi-step mathematical problems using rational numbers in any form. • convert between decimals		MP.1 MP.2 MP.3 MP.4 MP.5 MP.6

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	and fractions and apply properties of operations when calculating with rational numbers. • estimate to determine the reasonableness of answers.		
Use variables to represent quantities in a real-world or mathematical problem by constructing simple equations and inequalities to represent problems. Fluently solve equations; solve inequalities, graph the solution set of the inequality and interpret the solutions in the context of the problem (*Equations of the form px + q = r and p(x + q) = r and inequalities of the form px + q > r, px + q ≥r, px+ q ≤ r, or px + q < r, where p, q, and r are specific rational numbers).* 7.EE.B.4., 7.EE.B.4.b	**Students are able to:** • compare an arithmetic solution to a word problem to the algebraic solution of the word problem, identifying the sequence of operations in each solution. • write an equation of the form px + q = r or p(x + q)=r in order to solve a word problem. • fluently solve equations of the form px + q = r and p(x + q)= r. • write an inequality of the form *px + q > r, px + q < r* , *px + q ≥ r* or *px + q ≤ r* to solve a word problem. • graph the solution set of the inequality. • interpret the solution to an inequality in the context of the problem.	▪ Each task requires students to solve two equations (one of each of the given two forms). Only the answer is required. ▪ Comparison of an algebraic solution to an arithmetic solution is not assessed here; for this aspect of 7.EE.4a, see 7.C.5. ▪ Tasks may involve, ≤ or ≥	MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7
Apply the distributive property and collect like terms to solve linear equations in one variable	**Concept(s):** • Linear equations may have an infinite number of	▪ Most tasks involve contextual real-world word problems	MP.1. MP.2 MP.6

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that contain rational numbers as coefficients. Use an equivalent equation of the form x = a, a = a, or a = b (where a and b are different numbers) to describe the number of solutions. 8. EE.C.7, 8. EE.C.7a, 8.EE.C.7b	solutions. • Linear equations may have no solution or a single solution. **Students are able to:** • Give examples of linear equations in one variable with one solution (x = a), infinitely many solutions (a = a), or no solutions (a = b.) • Transform a given equation, using the properties of equality, into simpler forms. • Transform a given equation until an equivalent equation of the form x = a, a = a, or a = b results (a and b are different numbers). • Solve linear equations that have fractional coefficients; include equations requiring use of the distributive property and collecting like terms.	▪ Given an equation present the solution steps as a logical argument that concludes with the set of solutions (if any).	MP.7.

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VI. Differentiations /Modifications Teaching Strategies

Research Based Effective Teaching Strategies	**Modifications (how do I differentiate instruction?)**	**Special Education**	**Strategies for English Language Learners**
Task /Activities that solidifies mathematical concepts Use questioning techniques to facilitate learning Reinforcing Effort, Providing Recognition Practice, reinforce and connect to other ideas within mathematics Promotes linguistic and nonlinguistic representations Cooperative Learning Setting Objectives, Providing Feedback Varied opportunities for students to communicate mathematically Use technological and /or physical tools	**Modifications** Before or after school tutorial program Leveled rubrics Increased intervention Small groups Change in pace Calculators Extended time Alternative assessments Tiered activities/products Color coded notes Use of movements Use any form of technology ***Have students model finding a new amount after an increase and after a decrease on note cards for reference. Students should begin to recognize two methods for solving for percent of change: find an original amount, then add/subtract the change. another method is	Change in pace Calculators Alternative assessments Accommodations as per IEP Modifications as per IEP Use graphic organizer to clarify mathematical functions for students with processing and organizing difficulties'. Constant review of math concepts to strengthen understanding of prior concepts for difficulties recalling facts. Use self-regulations strategies for student to monitor and assess their thinking and performance for difficultly attending to task Cooperative learning (small group, teaming, peer assisted tutoring) to foster communication and strengthen confidence. Use technology and/or hands on devices to: clarify abstract concepts and process for: 1. Difficulty interpreting pictures	*Whiteboards* *Small Group / Triads* *Word Walls* *Partially Completed Solution* *Gestures* *Native Language Supports* *Pictures / Photos* *Partner Work* *Work Banks* *Teacher Modeling* *Math Journals* ***Simple problems involving ratios can often be express with pictures rather than words. Begin lessons with picture problems instead of word problems can help focus on the math concept. Students use their understanding of the mathematical concept to understand what is asked for and in the word problem. Another pictorial way to represent

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	multiplying the original amount by the quantity 100% plus or minus the percent change **Extension:** ***Using the internet, students research the sales tax structure of various states, sales tax and what items incur sales taxes. Students can also explore city and county sales and federal taxes that are paid on such items such as gasoline. Students research property tax of the town* ****Encourage students to create rate problem on*	and diagram. 2.difficulties with oral communications 3. Difficulty correctly identifying symbols of numeral 4.Difficulty maintaining attentions Simplify and reduces strategies / Goal structure to enhance motivation, foster independence and self-direction for: 1.Difficulty attending to task 2. Difficulty with following a sequence of steps to solution. 3.Difficulty processing and organizing Scaffolding math idea/concepts by guided practice and questioning strategies' to clarify and enhance understanding of	problems involving rates and ratios is using bar diagrams. *See Connected Mathematics Program 3 Classroom Differentiation for English Language Learners*

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21st Century Learning Skills: Teamwork and Collaboration Initiative and Leadership Curiosity and Imagination Innovation and Creativity Critical thinking and Problem Solving Flexibility and Adaptability Effective Oral and Written Communication Accessing and Analyzing Information	*topics of interest (ex. The number of songs on the ratio to the number of commercials with a hour)* *See Connected Mathematics Program 3 Classroom Differentiation for Gifted Students*	math big ideas for: 1.Difficulty with process and organization 2.Difficulty with oral and written communication Teacher models strategies' and think out aloud strategies to specify step by step process for: 1.Difficulties processing and organization 2. Difficulty attending to tasks. Use bold numbers and/or words to draw students' attention to important information. *See Connected Mathematics Program 3 Classroom Differentiation for Special Needs Students*	

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VII. Instructional Resources

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<tr><td rowspan="3">Short constructed responses
Extended responses
Checks for Understanding
Exit tickets
Teacher observation
Projects
Timed Practice Test – Multiple Choice & Open-Ended Questions

<u>Performance Tasks:</u>

<u>7.RP.A.2c Gym Membership Plans</u>

<u>8.EE.B.5 Who Has the Best Job?</u>
<u>8.EE.C.7 Coupon versus discount</u>

Additional Writing Tasks for Class Use:

<u>7.RP.A.1 Cooking with the Whole Cup</u>
<u>7.RP.A.2c Gym Membership Plans</u>
<u>7.EE.B.3 Discounted Books</u>
<u>7.EE.B.3 Shrinking</u>
<u>7.EE.B.4 Fishing Adventures 2</u>
<u>7.EE.B.4, 7.NS.A.1 Bookstore Account</u>
<u>7.EE.B.4b Sports Equipment Set</u>
<u>8.EE.C.7 The Sign of Solutions</u></td><td colspan="2">Connected Math Program Grade 7 Unit: Comparing and Scaling: Investigation 1-3
Connected Math Program Grade 7 Unit: Moving Straight Ahead: Investigation 1-3
Connected Math Program Grade 8 Unit: Thinking Mathematical Model: Linear and Inverse Variation: Investigation 1-2
<u>Scope and Sequence for Grade 7</u></td></tr>
<tr><td colspan="2">Technology</td></tr>
<tr><td>Resources for teachers
*<u>NJ CORE</u>
<u>Connected Math Project (Michigan State University)</u>
<u>My Pearson Training : Connected Math Program</u>
<u>Annenberg Learning : Insight into Algebra 1</u>
<u>National Council of Teachers of Mathematics</u>
<u>Mathematics Assessment Projects</u>
<u>Achieve the Core</u>
<u>Illustrative Mathematics</u>
<u>Mathematics Assessment Projects</u>
<u>Get the Math</u>
<u>Webmath.com</u>
<u>sosmath.com</u>
<u>Mathplanet.com</u>
<u>Interactive Mathematics.com</u>
<u>Inside Mathmatics.org</u>
<u>Asia Pacific Economic Cooperation :</u>
<u>:Lesson Study Videos</u>
<u>Genderchip.org</u>
<u>Interactive Geometry</u></td><td>Resources for Students
<u>My Math Universe.com</u>
<u>Math is Fun website</u>
<u>Khan Academy</u>
<u>Figure This.org website</u>
<u>Virtual Nerd website</u>
<u>Math Snacks websites</u>

<u>Internet 4 Classroom website</u>
<u>A Maths Dictionary for kids</u></td></tr>
</table>

<u>Hyperlinks are noted underlined in italics</u>

8.EE.C.7 Coupon versus discount 8.EE.C.8a Intersection of Two Lines 8.EE.C.8 How Many Solutions **Project** *Teach 21 Problem Based Learning Project : Techno Youth and Auto Fleet*	*Mathematical Association of America* *learner.org* *Math Forum : Teacher Place*	

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